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RESEARCH ARTICLE

HISTORY OF NEW WORLD SILVER PRODUCTION TRENDS BETWEEN 1521—1810

Mahashweta Das[1*], Malabika Ray[1], Anil Kumar Sarkar[2], Vasant Patel[3] and Dharmesh P. Raykundaliya[4]
[1]*Department of History, The University of Burdwan, Burdwan, West Bengal, India*
[2]*Department of History, University of Kalyani, Kalyani, Nadia, West Bengal, India*
[3]*P.G. Department of History, Sardar Patel University, Vallabh Vidyanagar, Anand, Gujarat, India*
[4]*P.G. Department of Statistics, Sardar Patel University, Vallabh Vidyanagar, Anand, Gujarat, India*
[*]*Corresponding author: mahashweta259@gmail.com*

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Abstract: In the world economy, silver has acted as a significant character as a universally expensive commodity and, in most countries, a currency. In early and modern economics, out of the most widely-traded commodities, silver is an exemplary commodity for which comprehensive documentation appears feasible. The early history of silver production data was not consistent, as they were recorded in many diaries, or books, based on the author's own estimation, or obtained from some unreliable sources. It is well-known that any historical data set is not accurate as physical measurement, or scientific experimental data. It is always necessary to study history with unbiased or robust historical data which can only be derived by using some scientific modeling method from the raw available data. The present paper focuses on studying the history of new world- Spanish and Portuguese colonies' silver production trends from 1521 to 1810 statistically, using probabilistic parametric and cubic spline models. The report not only derives the robust estimates of silver productions during this period, but also focuses on many historical events such as the early industrial advancement, industrial revolution, and depression status in silver mining during the period. All these above historical events are identified from the derived probabilistic parametric and cubic spline models. In addition, a probabilistic parametric model gives better estimates than a cubic spline model within this period.

Keywords: History of silver; Cubic spline; Joint generalized linear models; Silver production trend; Spanish and Portuguese colonies.

Introduction

Silver is one of the scarcest metals in nature and definitely among the most used ones, not only as a precious good like gold for financial investments but also for many industrial critical applications. The gold and silver history is approximately as long as the human civilization history. Silver and gold have been inextricably related to human civilizations since at least 6000 BC. Merely, there is no way of watching world history without encountering the silver and gold history. In ancient times, Egypt was the main gold-producing country (Habashi, 2014). Coptos town in Egypt, currently known as Quft on the eastern side of the Nile River, was the primary town of the Nomos of Harawi. In the eleventh dynasty (2133 - 1991 BC): Coptos was overshadowed by Thebes. Luxor city, the capital of the Middle Kingdom (2133 BC) of ancient Egypt, was located 50 km to the south of Coptos, which was the world's first gold boom town (Habashi, 2014).

Professor John TePaske (2010) mentioned in his book that the Spanish nation was driven by the three words such as God, glory and gold. The Spanish nation felt that their power and prestige could be increased by the leadership of supplying gold and silver over the world. This concept was the primary fuel for a nation to search for these metals all over the world. For the shortage of silver and gold in Europe, in the late fifteenth century, the outgoing nation-states like Spain and Portugal were compelled to inspire the search for new sources of silver and gold. From the diary of Pierre Vilar, it was noted that Columbus mentioned that gold was increased by sixty-five times between the period October, 1492 and January 1493 after the Genoan Born sailor returned to Castile (Vilar, 1991). He returned from his first voyage along with 33,100 silver *pesos* of eight *reales,* or gold nuggets costing 20,000 *escudos*, nearly 9,000,000 *maravedís.* It is noted that *Maravedís* small copper coins were mainly used in Castile. Actually, *maravedis* was considered a standard Spanish account unit. Note that a silver peso (or simply piece) consisted of eight *reales,* while a real was costing 34 *maravedis*, therefore, a peso was costing 272 *maravedis*. The primary purpose of Columbus' second voyage was to search with high priority the metals such as silver and gold along with the time of victory and discovery. He returned from his second voyage with 30,000 gold ducats worth 11,250,000 *maravedis* and 41,000 silver *pesos*. Columbus proved his statements that the Indies had provided new riches sources for the Catholic Kings (Jaime Vicens Vives, ed. 1961).

The conquistadores Spain and Portugal, settlers and mobsters who followed them collected a large quantity of gold and silver metals in the New World. Therefore, silver was subordinated in Spanish America. From the early time such as the colonial period, Europeans unfolded gold in the Caribbean, New Granada (currently known as Colombia), Chile, and Ecuador. These areas were known as the major producers of silvers and gold in Spanish America. Mainly, in the eighteenth century, the Portuguese eventually collected gold in great abundance in Luso-America (Brazil). During the three centuries of European domination, they made heroic efforts to account for New World silver and gold output, as big quantities of costly metals were turned out, minted and refined in the Indies by those periods.

For approximately three centuries in consequence of Brazil unfolded around 1500, the Portuguese judicature was overflowed with documents and reports of imaginary gold and silver turns in Brazil (Russell-Wood, 1984). The documents had generally lacked basis, a mixture of misguided faith, which were occupied as legends (or stories) of native American. The legends were a highly hopeful description for the investigators, and they had felt that the stories had correct reasons that a continent that had remunerated the Spaniards with silver, gold and emeralds must also occupy costly metals located to the Portuguese by the agreement of Tordesillas (1494). Note that whole these records were not completely free from truth. Actually costly metals such as gold, silver were obtained in Sao Vicente around the 1560s. Also around the1570s Paulistas found alluvial gold in Paranagua. There were records of a sudden discovery of a lot of gold by Joao Coelho de Sousa. His brother also obtained legal permission in 1584 to commence an expedition to establish the truth of discovering a lot of gold (Russell-Wood, 1984).

The eminent German scientist Alexander von Humboldt (1769 - 1859) visited extensively all over Spanish America at the end of the 18th century and beginning of the 19th. Based on the blessing of Charles IV, he collected the Spanish royal fiscal records, and published his details in the book entitled- *Political Essay on the Kingdom of New Spain.* In the book, the author not only reported his own New World silver and gold production estimates, in addition, he also presented the estimates of those who had been done before him. These works established that his estimates were the benchmarks for production of gold and silver for the New World (Humboldt, 1972). A Spanish jurist Juan Solorzano Pereira (1575 - 1655), revealed his book — *De indiarium jure* between 1629 and 1637, and later on it was revealed in five volumes as *Política Indiana,* where the author estimated New World bullion yields between 1492 and 1628 was approximately 1,500,000,000 silver *pesos* of 272 *maravedis* (Juan de Solorzano Pereira, 1648). In the eighteenth century two French researchers, Abbe Raynal or Guillaume Francois Thomas Raynal (1713 - 1796) and Jacques Necker (1732 - 1804) presented their predictions of New World silver and gold yields. Between 1492 and 1770, Raynal calculated that New World gold and silver yields were 5,154,000,000 *pesos* that was reported in the book *Histoire philosophique et politique des établissements et du commerce dans les deux Indes*, which was first published in six volumes in 1770 and in many later editions were published (Raynal et al, 2010). Jacques Necker, Finance Minister under Louis XVI confined his studies between the period 1763 and 1777, he thought that 304,000,000 *pesos* were yielded by the new world mines. One more French investigator Francisque Michel (Francisque Michel, 2018) estimated that new world gold and silver yields were 5,072,000,000 pesos between 1492 and 1775, which was very near to Raynal's estimate (Raynal et al, 2010).

In the beginning of eighteenth-century, economist and politician Geronimo de Uztariz (1670 - 1732) calculated Spanish American gold and silver yields from 1492 - 1724 were nearly 3,536,000,000 *pesos* that was reported in his book - *Theorica and practice of commerce and marine in different speeches* (or in Spanish- *Teórica y práctica de comercio, y de la marina, en diferentes discursos*) (Geronimo de Uztariz, 1724). Scottish historian William Robertson (1721 - 1793) calculated the Indies gold and silver yields from 1492 - 1775 were nearly 8,800,000,000 *pesos* that was very high according to Humboldt (Humboldt, 1972), as he studied the government given data, and long-range mining and minting situations. Following very closely the whole new world period (1492-1803) of Portuguese and Spanish domination, Humboldt calculated the silver and gold produced quantities in the Indies were nearly 5,706,700,000 *pesos*, colonial Spanish output nearly 4,851,156,000 pesos, and Luso-American yields nearly 855,544,000 *pesos*. In addition, he also tried to calculate for untaxed and unregistered yields. He predicted yield totals for each region as follows. In Luso-America it was 171,000,000 pesos, while in Spanish America it was 816,000,000 *pesos*. One German researcher Adolf Soetbeer (1814 - 1892) calculated the new world silver and gold output between 1493 and 1810 (Adolf Soetbeer, 1892). His calculations appeared in marks and kilograms of fine gold and silver, which were displayed in Table 1-1, page 19 (TePaske 2010). He estimated that within the period 1493—1810, Spanish and Portuguese America obtained nearly 126,657,400 and 3,743,770 kilograms of silver and gold, respectively.

The treasury account summaries for the Mexico City Caja had been published by TePaske in 1976 (TePaske, 1976). In the 1980s, TePaske and his co-researchers revealed similar accounts for some regions such as Peru, Chile, Upper Peru, other Mexican treasury offices, Ecuador (in 1990) (TePaske and Klein, 1982, 1986; TePaske and Jara, 1990). TePaske and his co-researchers also analyzed the data and interpreted that there was not any long depression in the mining industry of Mexico during the 17th century (TePaske and Klein, 1981). TePaske (1983) calculated the bullion flows issues from Spanish America to Europe and Asia during the 17th and 18th centuries. Focusing trends in expenditure and taxation in colonial Peru, Bolivia and Mexico during the 18th century had been presented in the book by Klein (1998). Prof. TePaske and his co-researchers cleaned and examined the silver and gold production data for long times, and finally published in 2007 in the website - *http://www.insidemydesk.com/hdd.html* and also in the book by TePaske (2010).

It is shown in the above that the reported Spanish and Portuguese colonies' silver production quantities between 1521 and 1810 were not consistent, and there were many discrepancies, missing and also unrecorded data. Prof. TePaske and his co-researchers cleaned the data using only the records and their personal intuitions. Best of our knowledge, little study has been made to clean the data adopting probabilistic modeling. The present report attempts to derive the new world silver production quantities between 1521 and 1810 using probabilistic modeling, based on the cleaned data provided by Prof TePaske (2010). Three different fitted trend curves (joint Gamma model, joint Log-normal and cubic spline) of the new world silver production quantities between 1521 and 1810 have been derived in the report. Fitting of the curve has been diagnosed using graphical analysis. Using the fitted trend curves, the robust estimates of the new world silver production quantities between 1521 and 1810 have been provided.

The report is structured as follows. Historical trend concept is presented in the next section, and the rest sections are materials and methods, statistical analysis results, and discussion and conclusion. It has been derived in the report that the response new world's silver production quantity is heterogeneous, and the fitted parametric mean trend curve is a non-linear polynomial of degree four of transformed time (t), and its variance function is nonlinear of four-degree of transformed time t. The cubic spline model has been developed, and it is found that the parametric model gives a better estimate than the cubic spline model within the range. Both the derived models can forecast the mean of the new world silver production quantity at any time during the period from 1521-1810, which are more robust and consistent than the recorded data.

Historical Data Trends Identification Method

Trend is associated with a long time period data set, which is termed as time series data that has four components such as Secular trend (or Trend) (T_t), Seasonal variation (S_t), Cyclical variation (C_t), and Irregular variation (I_t). Trend denotes a long term time series data changing representing smooth upward or downward movement. The periodic variation of a time series data with length period less than or equal to one year is known as seasonal variation, while the periodic variation having length period more than one year is known as cyclical variation. Any abrupt variation such as very low decrease or very high increase of time series data at a certain point of time due to some uncertain cause is known as Irregular variation. For more details readers may go through (Montgomery, Jennings and Kulachi, 2016; Shumway and Stoffer, 2017).

The present report is aimed for the logical problem of separating trends from a historical time series data set adopting the time series analysis. In addition, it tries to derive the best secular trend associated with a historical time series data set. One should study the secular trend problem as one of historical description. Based on this view, the trend problem is not different with respect to the general historical description problem. The historian tries to develop a trend from her/his mass of material, a descriptive substance which will bring out clearly the nature of the primary factors which have been at work.

Professor Schlesinger stated in the preface of his book entitled- *Political and Social History of the United States,* that constant stresses have been applied on the great dynamic currents which have shaped the nation's life. According to Professor Schlesinger these stresses were, and still are (i) the constant fighting for greater democracy; (ii) the nationality growth; (iii) the struggle for social advancement, including the children and women progress, the successive humanitarian reform activities and contest for free public schools; (iv) the national boundaries expansion; and (v) continuous production methods improvement and its distribution (adopting advance machinery and technology and the required social adjustments thereby). Similar to secular trends, these are all encountered as continuing and continuous processes (Schlesinger, 1926, p. viii).

The phenomenon of secular trend explains, it may be revealed, definitely a special case of historical illustration. The historical materials consist of a significant number of statistical series, which are provided by particular non-statistical information. Generally, the historical problem is to present the best illustration possible of the primary tendencies which these facts reveal. The statistician represents this historical description, not in words, but in probabilistic models, using lines or curves. Specifically, he/she presents it in terms of secular trends using probabilistic models, about these historical trends (Mills, 1932; Frickey, 1934). So, the probabilistic models, revealing lines, or curves labeled "secular trend" represents the use of someone's discretionary judgment.

In practice, the trend is a statistical term, which is generally expressed by a mathematical equation based on the original data recorded against time. There is a chance for contrary opinions with respect to the validity of the trend concept as a statistical statement. Note that such a statistical statement can have no value, or little unless the derivations are preceded by a comprehensive theoretical investigation, or through historical examination, or both, which should arise on the factors behind the time series movements, otherwise it should not be considered as the secular trend. In general, a mathematical curve option to present the primary trend is confined to functions such as the linear polynomial, simple logistic, logarithmic parabola, Gompertz equation, which satisfy the original data characteristic variations. For all the above trend functions, only the mean trend function is considered assuming that variance is constant, which may not always be true for historical data. Note that variance has its own interpretations, and it could occur due to many problems, which are also related to some historical events. For heterogeneous historical data, one should consider both mean and variance functions jointly (Lee, Nelder and Pawitan, 2017). This approach is very little discussed in statistical time series analysis (Montgomery, Jennings and Kulachi, 2016; Shumway and Stoffer, 2017).

Material and Statistical Methods

Materials

The article has developed Spanish and Portuguese colonies silver production trend for the period 1521-1810 adopting the data set by Late Professor John TePaske, who collected silver registrations data for those areas from treasury records. Professor John TePaske and his co-researchers started to reveal the data 33 years ago. After these publications Prof. TePaske extensively searched to clean the original silver production data series. Professor TePaske approved the posting of his revised silver and gold series here link — *http://www.insidemydesk.com/hdd.html* for Spanish, Portuguese, America and other scholars to use. For ready reference it is given in Table 1.

Statistical Methods

The present study considers the Spanish and Portuguese colonies silver yields trend between the period 1521 to 1810 using the registered data set as given in the above link. It is noted that the response to silver production quantity over the time is continuous, positive, heterogeneous and non-normally distributed. We fit both gamma and lognormal distributions to Silver Production Quantities in thousands, say SPQ1000. Shape and scale parameters of both gamma and lognormal distributions are obtained through moment matching estimation method. They are respectively given by (2.925774, 0.0000247) and (11.5345218, 0.5422218). Further, the p-value of Kolmogorov Smirnov Test (KS-test) corresponding to gamma distribution and lognormal distribution are 0.5716 and 0.4971 respectively. From the p-value of KS-test which is greater than 0.05, we can say gamma distribution fits better than lognormal distribution. Hence, the silver production trend can be derived using a suitable transformation of the response, if it is stabilized under this transformation. But the

Table 1: New World Silver Registrations Decennial in Pesos and Mean estimated Silver

Decade	*Silver Pesos (SP)*	*SP1000 (SP in thous-ands)*	*Mid period*	*t= (Mid period -1655.5) /10*	*Gamma fitted estimated SP (in thousands)*	*Lognormal fitted estimated SP (in thousands)*	*Spline estimated SP (in thousands)*
1521-1530	340000	340	1525.5	-13	3413.772	2830.59	-3049.6
1531-1540	7550000	7550	1535.5	-12	9196.126	8054.512	12252.8
1541-1550	28120000	28120	1545.5	-11	20185.47	18459.9	27555.2
1551-1560	42710000	42710	1555.5	-10	37082.01	35055.37	43380.4
1561-1570	56050000	56050	1565.5	-9	58464.48	56645.43	59438
1571-1580	71470000	71470	1575.5	-8	80989.09	79845.87	75293.2
1581-1590	100190000	100190	1585.5	-7	100752.3	100473.6	90389.6
1591-1600	113400000	113400	1595.5	-6	114855.8	115302.9	102916.8
1601-1610	121810000	121810	1605.5	-5	122231.5	123065.6	111610
1611-1620	124280000	124280	1615.5	-4	123508.2	124366.5	115953.2
1621-1630	123630000	123630	1625.5	-3	120317.1	120933	116025.2
1631-1640	128600000	128600	1635.5	-2	114552	114790.8	113066.4
1641-1650	102830000	102830	1645.5	-1	107878	107715.8	108221.2
1651-1660	92160000	92160	1655.5	0	101535.2	101016.9	103397.6
1661-1670	85730000	85730	1665.5	1	96347.41	95548.47	99203.6
1671-1680	100020000	100020	1675.5	2	92827.31	91830.22	96826.8
1681-1690	109850000	109850	1685.5	3	91302.79	90185.8	95891.6
1691-1700	92800000	92800	1695.5	4	92026.37	90862.06	96538.4
1701-1710	78250000	78250	1705.5	5	95256.58	94115.78	99792
1711-1720	92610000	92610	1715.5	6	101310.4	100268.3	107380.4
1721-1730	112450000	112450	1725.5	7	110585.4	109727.3	117865.3
1731-1740	130650000	130650	1735.5	8	123544.4	122966.4	132126.3
1741-1750	147940000	147940	1745.5	9	140641.2	140440.3	150440.7
1751-1760	174580000	174580	1755.5	10	162157.4	162400.2	171378.6
1761-1770	166720000	166720	1765.5	11	187911.6	188568.4	193411.6
1771-1780	216550000	216550	1775.5	12	216824.1	217660.3	217165.7
1781-1790	241880000	241880	1785.5	13	246394.3	246831.3	240964.2
1791-1800	289940000	289940	1795.5	14	272296.6	271304.7	264760.6
1801-1810	279460000	279460	1805.5	15	288501.8	284654.6	288556.9

variance of the response silver production quantity is not stabilized under any known transformation. So, it can be modeled by joint generalized linear models (JGLMs) using Log-normal or Gamma distribution which is illustrated in the book by Lee et al. (2017) and many research papers by Das and Lee (2009), Lesperance and Park (2003). For ready reference, it is very shortly given herein.

Joint lognormal generalized linear models: For a positive continuous dependent random variable Y_i's (here silver production quantities) with mean $E(Y_i) = \mu_i$ and $Var(Y_i) = \sigma_i^2 \mu_i^2 = \sigma_i^2 V(\mu_i)$ say, where σ_i^2 's are dispersion parameters and $V(.)$ reveals the dispersion function. In practice, the log transformation $Z_i = \log(Y_i)$ is frequently adopted to stabilize the variance $Var(Z_i) \approx \sigma_i^2$, but the variance may not always be stabilized. For obtaining a better model, JGLMs for the mean and dispersion can be considered. For log-normal distribution, JGLM of the mean and dispersion (response Y_i, with $Z_i = \log Y_i$) are expressed by

$$E(Z_i) = \mu_{zi} \text{ and } Var(Z_i) = \sigma_{zi}^2,$$

$$\mu_{zi} = x_i^t \beta \text{ and } \log(\sigma_{zi}^2) = g_i^t \gamma,$$

where x_i^t and g_i^t are the vectors of independent variables (here only time t with different degrees) associated with the regression coefficients β and γ, respectively.

Joint gamma generalized linear models: For the above Y_i's, the variance has two portions such as σ_i^2 (free of mean parameters μ_i's) and $V(\mu_i)$ (depending on the mean parameters). The dispersion function reveals the GLM family distributions. For instance, if $V(\mu) = \mu$, it is Poisson, normal if $V(\mu) = 1$, and Gamma if $V(\mu) = \mu^2$, etc. Gamma JGLMs mean and dispersion models are

$$\eta_i = g(\mu_i) = x_i^t \beta \text{ and } \varepsilon_i = h(\sigma_i^2) = w_i^t \gamma,$$

where $g(\cdot)$ and $h(\cdot)$ are known as GLM link functions connected to mean and dispersion predictors, and x_i^t, w_i^t are the explanatory time variable with different degrees linked to the mean and dispersion parameters respectively. Mean parameters are estimated by maximum likelihood (ML) method, and the dispersion parameters are estimated by restricted ML (REML) method (Lee et al. 2017).

Statistical and Graphical Analysis

The response decennial production of silver quantity has been modeled by JGLMs with both gamma and lognormal distributions. Here decennial production of silver quantity is treated as the dependent variable, and time t is considered as the independent variable. The best model has been received based on the lowest Akaike information criterion (AIC) value that lowers both the squared error loss and predicted additive errors (Hastie *et al.* 2009, p. 203-204). According to the AIC criterion, JGLMs lognormal (AIC= 613.3567) and gamma (AIC= 613.0128) fits are almost identical. In the mean model t, t^2, t^3 and t^4 are included due to functional marginality rule (i.e., if higher degree term is significant, then all its lower degree should be included) by McCullacgh and Nelder (1989). Also in the variance model, t, t^2, t^3 (even t^2 is insignificant) and t^4 are included due to functional marginality rule. Both lognormal and gamma JGLM analysis results are presented in Table 2. Both the models show very similar analysis.

Table 2: Silver production growth trend both for Gamma and Log-normal fit for the period 1521 to 1810

		Gamma fit				*Log-normal fit*			
		Estimate	*Standard error*	*t-value*	*P-value*	*Estimate*	*Standard error*	*t-value*	*P-value*
	Intercept	1.15300	0.0317	363.828	<0.0001	11.5200	0.0341	337.309	<0.0001
	t	-0.0578	0.0078	-7.371	<0.0001	-0.06.13	0.0086	-7.116	<0.0001
Mean	t^2	0.0041	0.0010	4.007	<0.0001	0.0043	0.0012	3.748	0.0002
Model	t^3	0.0013	0.0001	12.144	<0.0001	0.0014	0.0001	11.387	<0.0001
	t^4	-0.0001	0.0001	-8.231	<0.0001	-0.0001	0.0001	-7.646	<0.0001
	Intercept	-4.7218	0.5281	-8.941	<0.0001	-4.8096	0. 5292	-9.089	<0.0001
	t	0.2439	0.0889	2.742	0.0061	0.2791	0.0902	3.095	0.0019
Dispersion	t^2	-0.0145	0.0 161	-0.899	0.3686	-0.0131	0.0162	-0.807	0.4196
Model	t^3	-0.0029	0.0006	-4.325	<0.0001	-0.0034	0.0007	-4.745	<0.0001
	t^4	0.0002	0.0001	1.909	0.0562	0.0002	0.0001	1.985	0.0471
AIC		**613.0128**				**613.3567**			

The derived decennial production of silver quantity (Table 2) probabilistic joint gamma model is a data generated model, which is to be checked by model checking tools. Valid interpretations are always drawn from the data fitted probabilistic model. For the joint gamma-fitted decennial production of silver quantity models (Table 2), model checking using graphical analysis is shown in Figure 1. In Figure 1(a), absolute residuals are plotted against the fitted value, which is a closely flat straight line, concluding that variance is constant with the running means. Figure 1(b) presents a normal probability plot of the residuals of the gamma fitted mean model for silver production quantity (Table 2), which does not indicate any lack of fit.

Non-parametric function estimation method (Cubic Splines)

Non-parametric function estimation is well described in many statistical research articles by Ruppert, Wand, Carrol (2003); Green and Silverman (1994); Wahba (1990). Generally, for fitting trends, we use a known functional form known as a parametric model. However, we promote the smooth function even though its form is unknown, which is called a non-parametric function. In this article,

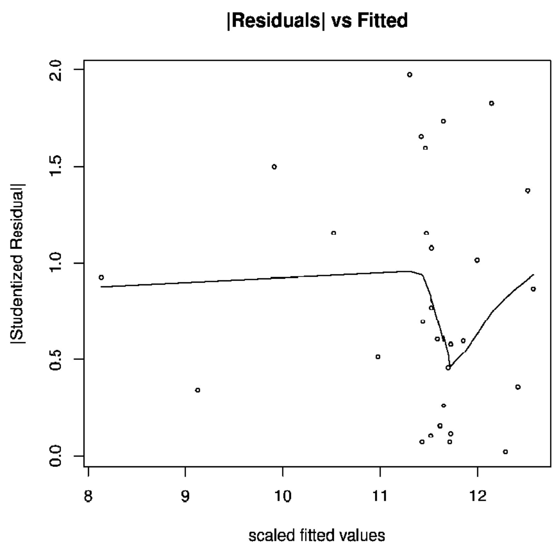

Figure 1(a)

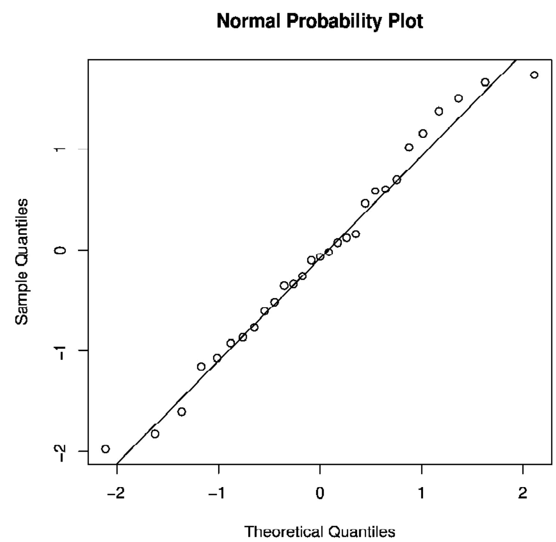

Figure 1(b)

Figure 1: For the joint Gamma fitted models of Silver production (Table 1), the (a) absolute student residuals plot with the fitted values, and (b) the normal probability plot for the mean model

we adopt a cubic spline as a non-parametric trend estimation method, and its fitting can be obtained using the R-package given in (Lee, Roonnegaard and Noh, 2017). For ready reference, all necessary R codes for distribution fittings, JGLMS and graphs are given in the Appendix. The analysis is performed using R.3.6.2 software.

Results

Table 2 shows the summarized results of decennial production of silver quantity under both lognormal and gamma model analysis. The gamma fitted mean model shows that the mean response of decennial production of silver quantity is fourth degree function of time "t". Note that time "t" is the transformed time, where t = (Mid period – 1655.5) /10 (shown in Table 1). In the mean model, t ($P<0.0001$), t^2 ($P<0.0001$), t^3 ($P<0.0001$) and t^4 ($P<0.0001$) are significant. On the other hand in the variance model, t ($P=0.0061$), t^3 ($P<0.0001$) and t^4 ($P=0.0562$) are significant but t^2 ($P=0.3686$) is not significant. Lognormal fit gives similar results (Table 2).

Gamma fitted silver production mean ($\hat{\mu}$) model (Table 1) is

$$\hat{\mu} = \exp.\ (1.153 - 0.0578t + 0.0041t^2 + 0.0013t^3 - 0.0001t^4),$$

and the gamma fitted silver production dispersion ($\hat{\sigma}^2$) model is

$$\hat{\sigma}^2 = \exp.\ (-4.7218 + 0.2439t - 0.0145t^2 - 0.0029t^3 + 0.0002t^4).$$

Lognormal fitted silver production mean ($\mu_{zi} = \hat{Z}$) model (Table 1) is

$\hat{Z}$ = 11.52 - 0.613 t + 0.0043 t^2 + 0.0014 t^3 - 0.0001 t^4, and lognormal fitted silver production dispersion ($\hat{\sigma}^2$) model is $\hat{\sigma}^2$ = exp(-4.8096 + 0.2791 t - 0.0131 t^2 - 0.0034 t^3 + 0.0002 t^4).

Discussions and Conclusions

The article has focused on the silver production trend of the new world from 1521 to 1810 using lognormal and gamma JGLMs, and cubic splines. It has been pointed out in the introduction section that silver production data regarding Spanish and Portuguese colonies are controversial, unrecorded and wrongly recorded. It is well known that the study of history is always information based. In addition, this information provides many economic, social and political status of the society during those periods, which are the main study subjects in history research. Generally, historical economists and historians have tried to study history with some maps, statistical figures, and graphs etc (TePaske, 2010). Best of our knowledge, there is very little history study adopting advanced probabilistic modeling. Therefore, the present outcomes are not compared with the previous similar studies, while the present outcomes can be compared with the previous historical recorded results in Table 1 and Figure 2.

Figure 2 presents the scattered plots of the original recorded data, and the three mean fitted curves such as gamma, lognormal polynomials and cubic spline, against the time. Gamma fitted values are closer to original points than the cubic spline and lognormal fit. At the initial and the end positions (boundary points), both the gamma & lognormal fitted polynomials, and also cubic spline trend curves are steady (Figure 2). Within the interval period and also at boundaries, the gamma fitted polynomial trend curve gives better estimates than cubic spline and lognormal models. In addition, both the gamma fitted polynomial and the cubic spline give efficient estimates in future and also in past. All these three mean fitted curves (Figure 3) show that silver productions were increasing from 1521 to 1620, and after that they were slightly fluctuating, or stable, or little decreasing up to around 1725, and after that they were continuously increasing up to 1810.

Figure 3 presents the silver production variance plot for the cubic spline which is increasing. It was observed from Figure 2 that silver productions were increasing from 1521 to 1620, and after that they were little fluctuating, and again they were increasing from 1730. So, these situations are reflected in the variance plot in Figure 3, where the slope of the smooth fitted line is increasing. At the beginning, it was almost stable up to around 1585, and after that it was steadily increasing up to around 1725, and after that it was highly increasing up to 1810. This implies that some advanced mining techniques were invented around the end of the seventeenth century, which were greatly applied in mining.

Industrial improvement had been initiated at the end of the seventeenth century in America and Europe. The discovery of the steam pump by Thomas Savery in 1698 and the Newcomen steam engine in 1712 greatly facilitated the drainage of water and mud, and enabled holes to be made deeper, enabling more silver and gold to be extracted. These engines and pumps were highly used in silver, or gold, or coal mining. These advancements had begun before the industrial revolution, which was known as the **First Industrial Revolution**, and it was the transition to new manufacturing processes in Europe and the United States, in the mid of eighteenth century. It is known historically that at the end of the seventeenth century there was a high advancement in industrial revolution in Europe and America. These situations are revealed in the present analysis. Moreover, current analysis does not show any depression in silver mining during the period 1521 to 1810, which is also supported by (TePaske and Klein, 1981). Thus, the current statistical analysis not only predicts the silver production quantity, but also reveals many historical facts of society such as silver production, depression, industrial revolution etc.

Our research has mainly two purposes. The first is to compare our predicted outcomes to those of previous recorded research. A second purpose was to evaluate the statistical assumptions followed by the response to silver production quantities. So, we have estimated and compared two models: a gamma model and a lognormal model. The present findings, though not completely conclusive, are revealing:

- The current outcomes are very close to 92% cases to the recorded cleaned data (Figure 2).
- Historical silver production trend has been expressed using a mathematical curve.
- Accepted models are verified by both mean and dispersion plots.

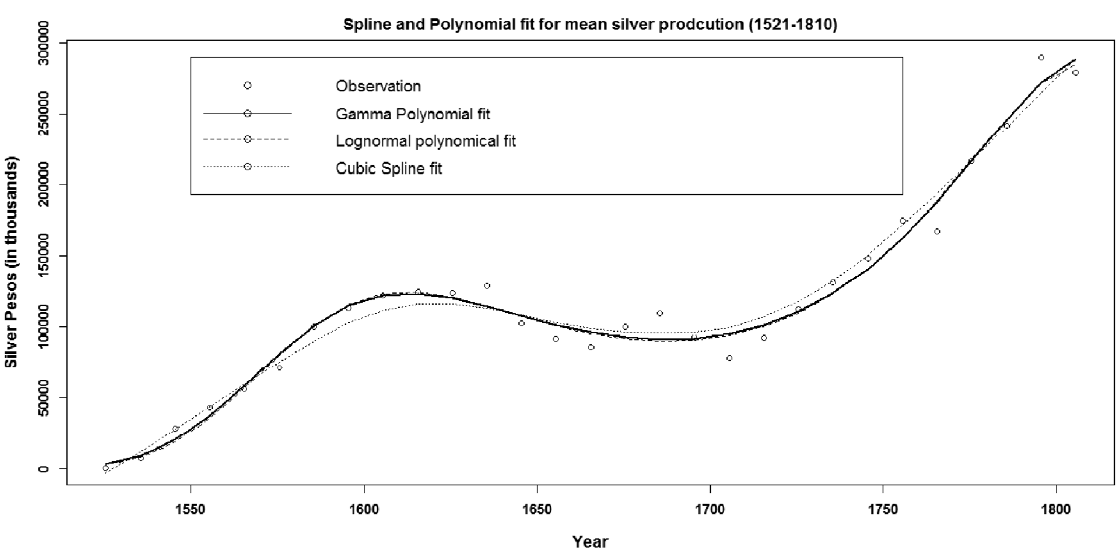

Figure 2: Scattered plot of the original observations and the smooth fitted mean trend curves for polynomial and cubic spline

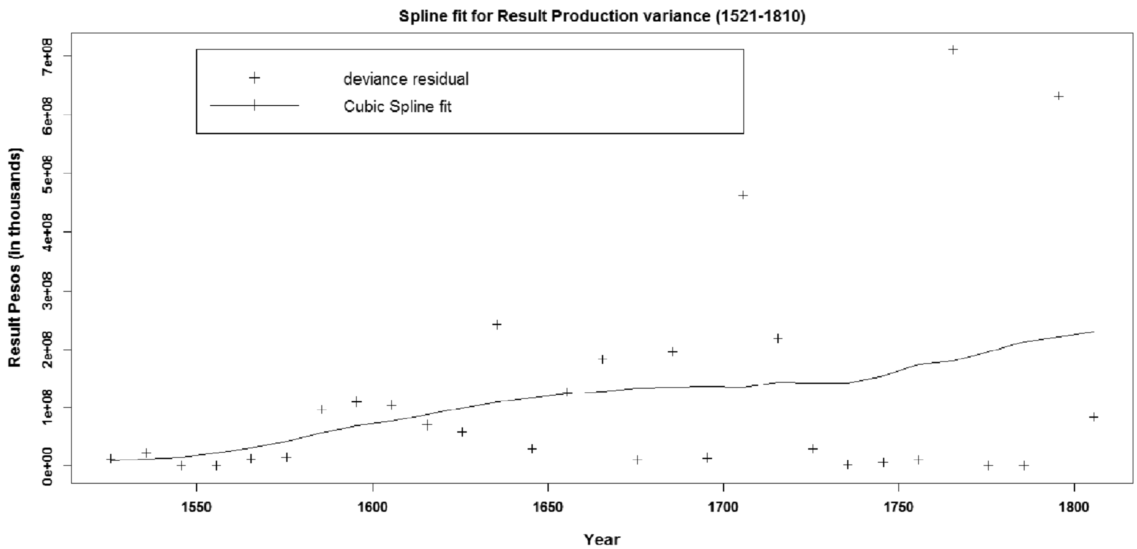

Figure 3: Scattered plot of the deviance residuals and dispersion plot cubic spline

Two main conclusions can be drawn from the present research. First, in order to reduce the controversy, unrecorded and wrongly recorded historical data, statistical modeling can be more effective. A second conclusion has to do with the use of statistical models. While further research is called for, we find that joint lognormal and gamma models give similar analysis as the AIC difference is less than one, which is insignificant. Moreover, these are much more effective than traditional linear polynomials, or the Gompertz equation, lognormal (with constant variance), or the logarithmic parabola, as they better fit the data. In short, researchers should have greater faith in these results than those emanating from lognormal (with constant variance) and other models.

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Appendix

R code

A. Gamma and lognormal distribution fittings

```
data=read.csv(file.choose())
names(data)
library(fitdistrplus)
hist(SP1000)
fitdist(data$SPQ1000,”gamma”,method=”mme”)
ks.test(data$SPQ1000,”pgamma”,2.925774,0.0000247)
fitdist(data$SPQ1000,”lnorm”,method=”mme”)
ks.test(data$SPQ1000,”plnorm”,11.5345218,0.5422218)
```

B. JGLMs for gamma and lognormal

```
data$t2=data$t^2
data$t3=data$t^3
data$t4=data$t^4

library(hglm)
```

```
library(dhglm)
#### Gamma Model
MM1<- DHGLMMODELING(Model=”mean”, LinPred=SPQ1000~ t+t2+t3+t4, Link=”log”)
DM1<-DHGLMMODELING(Model=”dispersion”, LinPred=phi~1+t+t2+t3+t4)
MODEL1<-dhglmfit(RespDist=”gamma”,DataMain=data, MeanModel = MM1,DispersionModel = DM1)
plotdhglm(MODEL1)
plotdhglm(MODEL1,type=”phi”)

#:::::::::LOG-NORMAL MODEL::::::::
data$logSPQ1000<-log(data$SPQ1000)
MM2<- DHGLMMODELING(Model=”mean”, LinPred=logSPQ1000~t+t2+t3+t4, Link=”identity”)
DM2<-DHGLMMODELING(Model=”dispersion”, LinPred=phi~1+t+t2+t3+t4)
MODEL2<-dhglmfit(RespDist=”gaussian”,DataMain=data, MeanModel = MM2,
                    DispersionModel = DM2)
plotdhglm(MODEL2)
plotdhglm(MODEL2,type=”phi”)
#### Predicted values
SPQ1000.gamma.fit=MODEL1$mu
SPQ1000.lognor.fit=exp(MODEL2$mu)
### AIC values
SPQ1000.gamma.aic=MODEL1$caic
SPQ1000.lognor.aic=MODEL2$caic
####C. For Graphs
data1=read.csv(file.choose())
names(data1)
attach(data1)
###Figure 2
plot(data1$year,data1$y, pch=1,xlab=””,ylab=””)
axis(1,font=2)
axis(2,font=2)
mtext(side=1,line=3,”Year”,font=2,cex=1.2)
mtext(side=2,line=3, “Silver Pesos (in thousands)”,font = 2,cex=1.2)
mtext(side=3,line=.5,”Spline and Polynomial fit for mean silver prodcution (1521-1810)”,font=2,cex=1.2)
lines(data1$year,data1$y_fitted, col=”black”, type=”l”, lty=1,lwd=2)
lines(data1$year,data1$y_lognormal, col=”black”, type=”l”, lty=2,lwd=1.5)
lines(data1$year,data1$y_spline,col=”black”,type=”l”,lty=3)
```

```
legend(1550, max(data1$y), legend=c("Observation","Gamma Polynomial fit","Lognormal polynomical fit"
 ,"Cubic Spline fit"), col=c( "black","black","black", "black"),pch=1,lty=0:3, cex=1.2)
##Figure 3
plot(data1$year,data1$d_spline, pch=3,xlab = "",ylab = "")
axis(1,font=2)
axis(2,font=2)
mtext(side=1,line=3,"Year",font=2,cex=1.2)
mtext(side=2,line=3, "Result Pesos (in thousands)",font = 2,cex=1.2)
mtext(side=3,line=.5,"Spline fit for Result Production variance (1521-1810)" ,font=2,cex=1.2)
lines(data1$year,data1$phi_spline, col="black", type="l", lty=1,lwd=1)
legend(1550, max(data1$d_spline), legend=c( "deviance residual", "Cubic Spline fit"),
 col=c( "black","black"),pch=3,lty=0:1, lwd=1,cex=1.2)
```